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INFLUENCE OF THE MOBILITY OF A BOUNDARY ON THE TEMPERATURE FIELD OF A HALF-SPACE UNDER UNSTABLE CONDITIONS OF HEAT EXCHANGE WITH THE ENVIRONMENT

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Using mathematical-simulation methods, we have investigated the special features of the process of formation of the temperature field in a half-space with a boundary moving following the prescribed law in the case of realization of unstable operating conditions of heat exchange with the environment which lead to the time dependence of the heat-transfer coefficient.

Solution of many practically important problems is associated with the need for mathematical simulation of the processes of heat transfer in solids when the conditions of unsteady heat exchange with the environment, resulting in the time change of the heat-transfer coefficient, are realized [1–5]. The difficulties arising in solving such problems are well known [2] and they become even more aggravated in the cases where account must be taken of the influence of different mechanical and physical-chemical processes on the temperature field of a thermally loaded region. The occurrence of these processes inevitably leads to a change in the dimensions of a solid because of the time change in position of its boundaries.

Among the problems of nonstationary heat conduction in regions with moving boundaries, of special interest are problems associated with investigation of the temperature fields in regions with boundaries moving following a prescribed law [3, 4].

The main aim of the investigations performed is to study the features of the process of formation of the temperature field of a solid simulated by a half-space with a boundary moving following the known law $l = l(\mathrm{Fo})$ under unstable conditions of heat exchange with the environment.

The object of the investigations was the one-dimensional mathematical model of the process under study:

$$\frac{\partial \theta(\xi, \mathrm{Fo})}{\partial \mathrm{Fo}} = \frac{\partial^2 \theta(\xi, \mathrm{Fo})}{\partial \xi^2}\,; \quad \xi > l(\mathrm{Fo})\,, \quad \mathrm{Fo} > 0\,, \quad \theta(\xi, 0) = 0\,,$$
$$\left.\frac{\partial \theta(\xi, \mathrm{Fo})}{\partial \xi}\right|_{\xi = l(\mathrm{Fo})+0} = \mathrm{Bi}(\mathrm{Fo})\,[\theta(l(\mathrm{Fo}) + 0, \mathrm{Fo}) - 1]\,, \tag{1}$$

where $\theta(\xi, \mathrm{Fo}) \in L^2[l(\mathrm{Fo}); +\infty)$, i.e., at each fixed $\mathrm{Fo} \geq 0$ the function $\theta(\xi, \mathrm{Fo})$ is quadratically integrable with respect to the spatial variable $\xi \in [l(\mathrm{Fo}); +\infty)$, $l = l(\mathrm{Fo}) \geq 0$ is the known law of motion of the boundary,

$$\xi = \frac{x}{x_*}\,, \quad \mathrm{Fo} = \frac{\kappa \bar{t}}{x_*^2}\,, \quad \theta = \frac{T - T_0}{T_{\mathrm{env}} - T_0}\,, \quad \mathrm{Bi} = \frac{\alpha}{\lambda} x_*\,. \tag{2}$$

The main aim of the present investigations can be achieved by assuming that $l = l(\mathrm{Fo})$ is a nondecreasing, nonnegative function differentiable at least in a generalized sense, $l(0) = 0$, while the function $\mathrm{Bi} = \mathrm{Bi}(\mathrm{Fo})$ defined in

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Eq. (2) and characterizing the instability of the realized conditions of heat exchange with the environment satisfies the standard requirements of the theorem of existence and uniqueness of the solution of the considered problem [6]. In this case, it is allowable to use a moving coordinate system

$$X = \xi - l\,(\mathrm{Fo})\,, \quad t = \mathrm{Fo}\,, \tag{3}$$

in which the mathematical model (1) can be represented in the following form:

$$\frac{\partial\theta\,(X, t)}{\partial t} = \frac{\partial^2\theta\,(X, t)}{\partial X^2} + l'\,(t)\,\frac{\partial\theta\,(X, t)}{\partial X}\,; \quad X > 0\,, \quad t > 0\,, \tag{4}$$

$$\theta\,(X, t)\,\big|_{t=0} = 0\,, \tag{5}$$

$$\left.\frac{\partial\theta\,(X, t)}{\partial X}\right|_{X=0} = \mathrm{Bi}\,(t)\,[\theta\,(X, t)\,\big|_{X=0} - 1]\,, \tag{6}$$

$$\theta\,(X, t)\,\big|_{t\geq 0} \in L^2\,[0; +\infty)\,. \tag{7}$$

We suppose next that

$$\begin{gathered} Q_{\mathrm{C}}\,(p, t) = \Phi_{\mathrm{C}}\,[\theta\,(X, t)] \equiv \int_0^\infty \theta\,(X, t)\cos\,(pX)\,dX\,, \\ Q_{\mathrm{S}}\,(p, t) = \Phi_{\mathrm{S}}\,[\theta\,(X, t)] \equiv \int_0^\infty \theta\,(X, t)\sin\,(pX)\,dX \end{gathered} \tag{8}$$

are the representations of the cosine and sine Fourier transforms [7] over the spatial variable X with parameter p. Having successively applied the operators Φ_{C} and Φ_{S} to Eq. (4) and to initial condition (5), with account for Eq. (7) and boundary condition (6) we come to the following Cauchy problem:

$$\frac{dQ\,(p, t)}{dt} = A\,(p, t)\,Q\,(p, t) + f\,(t) + F\,(p, t)\,\theta\,(0, t)\,, \quad t > 0\,; \quad Q\,(p, 0) = \theta_2\,, \tag{9}$$

where

$$\begin{gathered} Q\,(p, t) = \begin{bmatrix} Q_{\mathrm{C}}\,(p, t) \\ Q_{\mathrm{S}}\,(p, t) \end{bmatrix}, \quad A\,(p, t) = \begin{bmatrix} -p^2 & pl'\,(t) \\ -pl'\,(t) & -p^2 \end{bmatrix}, \\ f\,(t) = \begin{bmatrix} \mathrm{Bi}\,(t) \\ 0 \end{bmatrix}, \quad F\,(p, t) = \begin{bmatrix} -\mathrm{Bi}\,(t) - l'\,(t) \\ p \end{bmatrix}, \quad \theta_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \end{gathered} \tag{10}$$

According to Eqs. (9) and (10), the representations $Q_{\mathrm{C}}(p, t)$ and $Q_{\mathrm{S}}(p, t)$ of the unknown temperature field $\theta(X, t)$ are completely determined by its values $\theta(0, t)$ on the boundary of the half-space $X \geq 0$, the velocity $l'(t)$ of motion of the boundary, and the function $\mathrm{Bi}(t)$.

If $R(t, \tau)$ is the resolvent of the Cauchy problem (9) and (10), its solution can be represented in standard form [8]:

$$Q(p,t) = \int_0^t R(t,\tau)\left\{f(\tau) + F(p,\tau)\,\theta(0,\tau)\right\} d\tau\,. \tag{11}$$

Since, in conformity with Eq. (10), the matrix functions $A(p, t)$ and

$$\int_\tau^t A(p,s)\,ds = \begin{bmatrix} -p^2(t-\tau) & p\left\{l(t) - l(\tau)\right\} \\ -p\left\{l(t) - l(\tau)\right\} & -p^2(t-\tau) \end{bmatrix}, \quad 0 \le \tau \le t$$

are commutative relative to the operation of multiplication, which can be checked immediately, we have [8]

$$R(t,\tau) = \exp\left\{\int_\tau^t A(p,s)\,ds\right\}. \tag{12}$$

According to the Cayley–Hamilton theorem [8], for any square matrix B of order n the characteristic polynomial $q(\mu)$ is also its annihilator polynomial, i.e., if

$$q(\mu) = \det(B - \mu I_n) \equiv \sum_{k=0}^{n} q_k\,\mu^k\,, \quad \text{then} \quad q(B) = \sum_{k=0}^{n} q_k B^k = \theta_n\,,$$

where $\left\{q_k\right\}_{k=0}^{n}$ are the coefficients of the characteristic polynomial; $B^0 = I_n$ and θ_n are the unit and square zero matrices of order n. Thus, any degree of the square matrix B of order n is representable in the form of a linear combination of square matrices $\left\{B^k\right\}_{k=0}^{n-1}$. With this result taken into account it is proved [8] that the matrix function $f(B)$ is a linear combination of square matrices $\left\{B^k\right\}_{k=0}^{n-1}$:

$$f(B) = \sum_{k=0}^{n-1} \alpha_k B^k\,,$$

where $\left\{\alpha_k\right\}_{k=0}^{n-1}$ are the unknown coefficients satisfying the system of linear algebraic equations

$$\Sigma\ \alpha_k\,\mu_j^k = f(\mu_j)\,, \quad j = 1, ..., n\,,$$

provided that the eigenvalues $\left\{\mu_k\right\}_{k=0}^{n}$ of the matrix B are different.

In the case under consideration, $n = 2$ and the matrix exponential function on the right-hand side of equality (12) is defined as

$$\exp\left\{\int_\tau^t A(p,s)\,ds\right\} = \alpha_0 I_2 + \alpha_1 \int_\tau^t A(p,s)\,ds\,.$$

The coefficients α_0 and α_1 are found in the following manner. Solving the characteristic equation

$$\det\left(\int_\tau^t A(p,s)\,ds - \mu I_2\right) = 0\,,$$

we calculate the eigenvalues

$$\mu_1 = -p^2 (t-\tau) + ip \left\{ l(t) - l(\tau) \right\}, \quad \mu_2 = -p^2 (t-\tau) - ip \left\{ l(t) - l(\tau) \right\}.$$

Then we write a system of linear algebraic equations relative to the coefficients α_0 and α_1,

$$\alpha_0 + \alpha_1 \mu_1 = \exp \left\{ \mu_1 \right\}, \quad \alpha_0 + \alpha_1 \mu_2 = \exp \left\{ \mu_2 \right\}$$

and obtain

$$\alpha_0 = \alpha_1 p^2 (t-\tau) + \exp \left\{ -p^2 (t-\tau) \right\} \cos \left\{ p \left[l(t) - l(\tau) \right] \right\},$$

$$\alpha_1 = \frac{1}{p \left[l(t) - l(\tau) \right]} \exp \left\{ -p^2 (t-\tau) \right\} \sin \left\{ p \left[l(t) - l(\tau) \right] \right\}.$$

Thus, with account for Eq. (12), the resolvent of the Cauchy problem (9) and (10) is

$$R(t, \tau) = \begin{bmatrix} \cos \left\{ p \left[l(t) - l(\tau) \right] \right\} & \sin \left\{ p \left[l(t) - l(\tau) \right] \right\} \\ -\sin \left\{ p \left[l(t) - l(\tau) \right] \right\} & \cos \left\{ p \left[l(t) - l(\tau) \right] \right\} \end{bmatrix} \exp \left\{ -p^2 (t-\tau) \right\} \tag{13}$$

and, according to Eqs. (11), (13), and (10), we have

$$Q_C(p, t) = \int_0^t \exp \left\{ -p^2 (t-\tau) \right\} \cos \left\{ p \left[l(t) - l(\tau) \right] \right\} \mathrm{Bi}(\tau)\, d\tau + \int_0^t \exp \left\{ -p^2 (t-\tau) \right\}$$

$$\left\{ p \sin \left\{ p \left[l(t) - l(\tau) \right] \right\} - \left[\mathrm{Bi}(\tau) + l'(\tau) \right] \cos \left\{ p \left[l(t) - l(\tau) \right] \right\} \right\} \theta(0, \tau)\, d\tau. \tag{14}$$

Using representation (14), the inversion formula of the cosine Fourier transform [7], and changing the order of integration, we obtain

$$\theta(X, t) = \frac{2}{\pi} \int_0^t \left\{ \int_0^\infty \exp \left\{ -p^2 (t-\tau) \right\} \cos \left\{ p \left[l(t) - l(\tau) \right] \right\} \cos (pX)\, dp \right\} \mathrm{Bi}(\tau)\, d\tau +$$

$$+ \frac{2}{\pi} \int_0^t \left\{ \int_0^\infty \exp \left\{ -p^2 (t-\tau) \right\} \Big[p \sin \left\{ p \left[l(t) - l(\tau) \right] \right\} \cos (pX) - \right.$$

$$\left. - (\mathrm{Bi}(\tau) + l'(\tau)) \cos \left\{ p \left[l(t) - l(\tau) \right] \right\} \cos (pX) \Big] dp \right\} \theta(0, \tau)\, d\tau. \tag{15}$$

Having calculated the internal improper integrals on the right-hand side of equality (15), we represent it as follows:

$$\theta(X, t) = \int_0^t K(t, \tau, X)\, \theta(0, \tau)\, d\tau + \int_0^t \Psi(t, \tau, X)\, \mathrm{Bi}(\tau)\, d\tau; \quad X \ge 0, \quad t \ge 0, \tag{16}$$

where

$$K(t,\tau,X)=\frac{1}{2\sqrt{\pi(t-\tau)}}\left\{\frac{1}{2(t-\tau)}\left[(l(t)-l(\tau)-X)\exp\left\{-\frac{[l(t)-l(\tau)-X]^2}{4(t-\tau)}\right\}+\right.\right.$$

$$\left.+(l(t)-l(\tau)+X)\exp\left\{-\frac{[l(t)-l(\tau)+X]^2}{4(t-\tau)}\right\}\right]-(\mathrm{Bi}(\tau)+l'(\tau))\times$$

$$\left.\times\left[\exp\left\{-\frac{[l(t)-l(\tau)-X]^2}{4(t-\tau)}\right\}+\exp\left\{-\frac{[l(t)-l(\tau)+X]^2}{4(t-\tau)}\right\}\right]\right\};\tag{17}$$

$$\Psi(t,\tau,X)=\frac{1}{2\sqrt{\pi(t-\tau)}}\left[\exp\left\{-\frac{[l(t)-l(\tau)-X]^2}{4(t-\tau)}\right\}+\exp\left\{-\frac{[l(t)-l(\tau)+X]^2}{4(t-\tau)}\right\}\right].\tag{18}$$

Equalities (16)–(18) determine uniquely the temperature field $\theta(X, t)$ of the half-space in the moving coordinate system (3) with the known law $\theta(0, t)$ of variation of the temperature on the boundary of this half-space.

Using expressions (17) and (18), we set

$$\varphi(t)=\int_0^t\Psi(t,\tau,0)\,\mathrm{Bi}(\tau)\,d\tau=\int_0^t\frac{1}{\sqrt{\pi(t-\tau)}}\exp\left\{-\frac{[l(t)-l(\tau)]^2}{4(t-\tau)}\right\}\mathrm{Bi}(\tau)\,d\tau\,,$$

$$k(t,\tau)=K(t,\tau,0)=\frac{A(t,\tau)}{\sqrt{t-\tau}}\,,\tag{19}$$

$$A(t,\tau)=\frac{1}{\sqrt{\pi}}\left[\frac{l(t)-l(\tau)}{2(t-\tau)}-\mathrm{Bi}(\tau)-l'(\tau)\right]\exp\left\{-\frac{[l(t)-l(\tau)]^2}{4(t-\tau)}\right\},\quad u(t)=\theta(0,t)\,.$$

In this case, in accordance with Eqs. (16) and (19), the function $u(t)$, determining the temperature of the boundary $X = 0$ of the considered half-space, is the solution of the Volterra integral equation of the second kind

$$u(t)=\int_0^t k(t,\tau)\,u(\tau)\,d\tau+\varphi(t)\,,\tag{20}$$

whose kernel $k(t, \tau) = (t-\tau)^{-1/2}A(t, \tau)$ has a weak singularity and is not a Fredholm kernel [9]. Nonetheless, it can be shown that, under the assumption of a piecewise continuity of the functions $l'(t)$ and $\mathrm{Bi}(t)$ on the segment $[0; t_*]$, where $t_* < +\infty$, the solution $u(t)$, $0 \le t \le t_*$, of the integral equation (20) exists and is unique in any of the functional spaces $C[0; t_*]$ and $L^p[0; t_*]$, $p > 2$ [9].

For numerical solution of the integral equation (20) we choose the natural number N, set the tabulation step s_u of the function $u(t)$ equal to

$$s_u=(N+1)^{-1}t_*\,;\quad t_i=is_u\,,\quad i=0,\ldots,N\,,\tag{21}$$

and construct an iterative process which makes it possible to calculate with a prescribed accuracy the value of $u(t_{m+1})$ from the known values of $\{u(t_i)\}_{i=1}^m$ of the sought solution $u(t) = \theta(0, t)$. Here $t_0 = 0$ and $u(0) = 0$.

We suppose next that

$$u_i=u(t_i)\,;\quad\varphi_i=\varphi(t_i)\,,\quad i=0,\ldots,N\,.\tag{22}$$

Then, according to Eqs. (19) and (20),

$$u_{m+1} = \int_0^{t_{m+1}} \frac{A(t_{m+1}, \tau)}{\sqrt{t_{m+1} - \tau}} u(\tau)\, d\tau + \varphi_{m+1} . \tag{23}$$

The integral on the right-hand side of equality (23) is improper, thus making it impossible to use immediately the known quadrature formulas [10, 11]. To overcome this difficulty, we set

$$J_{m+1} = \int_0^{t_{m+1}} \frac{A(t_{m+1}, \tau)}{\sqrt{t_{m+1} - \tau}} u(\tau)\, d\tau = J_{m+1}^{(1)} + J_{m+1}^{(2)} , \quad J_{m+1}^{(1)} = \int_0^{t_m} \frac{A(t_{m+1}, \tau)}{\sqrt{t_{m+1} - \tau}} u(\tau)\, d\tau ,$$

$$J_{m+1}^{(2)} = \int_{t_m}^{t_{m+1}} \frac{A(t_{m+1}, \tau)}{\sqrt{t_{m+1} - \tau}} u(\tau)\, d\tau , \tag{24}$$

where the integral $J_{m+1}^{(1)}$ is proper and its value can be calculated by using the known quadrature formula of trapezoids [11] and equalities (21) and (22):

$$J_{m+1}^{(1)} = \frac{\sqrt{s_u}}{2} \left[2 \sum_{i=1}^{m-2} \frac{A(t_{m+1}, t_i)}{\sqrt{m - i + 1}} u_i + A(t_{m+1}, t_m)\, u_m \right] . \tag{25}$$

Replacing the integration variable by $\tau = t_{m+1} - y^2$, we reduce the improper integral $J_{m+1}^{(2)}$ determined in Eq. (24) to the integral of the function having a removable discontinuity. Therefore, with account for Eqs. (21) and (22), we have

$$J_{m+1}^{(2)} = 2 \int_0^{\sqrt{s_u}} A(t_{m+1}, t_{m+1} - y^2)\, u(t_{m+1} - y^2)\, dy = \sqrt{s_u}\, [A(t_{m+1}, t_{m+1})\, u_{m+1} + A(t_{m+1}, t_m)\, u_m] , \tag{26}$$

where, according to Eq. (19),

$$A(t, t) = \lim_{\tau \to t-0} A(t, \tau) = -\frac{1}{2\sqrt{\pi}} [l'(t - 0) + 2\,\mathrm{Bi}\,(t - 0)] . \tag{27}$$

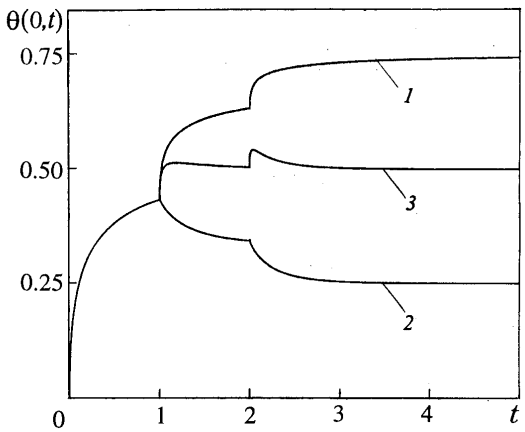
Having substituted the right-hand sides of equalities (24)–(27) in Eq. (23) and having expressed u_{m+1} from the relation obtained, we come to the following computational scheme for solution of the integral equation (20):

$$u_0 = 0 , \quad u_1 = \frac{\varphi_1}{1 - \sqrt{s_u}\, A(t_1, t_1)} ,$$

$$\begin{gathered} u_{m+1} = \frac{\varphi_{m+1}}{1 - \sqrt{s_u}\, A(t_{m+1}, t_{m+1})} + \frac{\sqrt{s_u}}{1 - \sqrt{s_u}\, A(t_{m+1}, t_{m+1})} \times \\ \times \left\{ \frac{3}{2} A(t_{m+1}, t_m)\, u_m + \sum_{i=1}^{m-1} \frac{A(t_{m+1}, t_i)}{\sqrt{m - i + 1}} u_i \right\} , \quad m = 1, ..., N . \end{gathered} \tag{28}$$

It should be noted here that equalities (28) have meaning only on satisfaction of the condition

$$s_u < \frac{1}{\max\limits_{0 \le t \le t_*} [A(t, t)]^2} = \frac{4\pi}{\max\limits_{0 \le t \le t_*} [l'(t - 0) + 2\,\mathrm{Bi}\,(t - 0)]^2} . \tag{29}$$



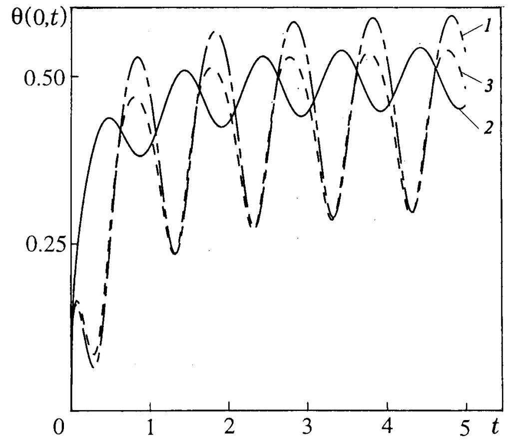


Fig. 1. Temperature of the half-space boundary $X = 0$ with piecewise-constant laws of variation of $\mathrm{Bi}(t)$ and $l'(t)$: 1) $\mathrm{Bi}(t) = [J(t) - J(t-1)] + 2[J(t-1) - J(t-2)] + 3J(t-2)$ and $l'(t) \equiv 1$; 2) $\mathrm{Bi}(t) \equiv 1$ and $l'(t) = [J(t) - J(t-1)] + 2[J(t-1) - J(t-2)] + 3J(t-2)$; 3) $\mathrm{Bi}(t) = l'(t) = [J(t) - J(t-1)] + 2[J(t-1) - J(t-2)] + 3J(t-2)$.

Fig. 2. Temperature of the half-space boundary $X = 0$ under different conditions of heat exchange with the environment and with different laws of variation of $l'(t)$: 1) $\mathrm{Bi}(t) = 1 - \sin(2\pi t)$, $l'(t) \equiv 1$; 2) $\mathrm{Bi}(t) \equiv 1$, $l'(t) = 1 - \sin(2\pi t)$; 3) $\mathrm{Bi}(t) = l'(t) = 1 - \sin(2\pi t)$.

The quantity on the right-hand side of Eq. (29) depends only on the characteristics determining the rate of the process studied and can be considered as an analog of the Courant parameter [12].

Let us consider some results of investigations reflecting the most characteristic features of the process of formation of the temperature profile $\theta(0, t)$ on the half-space boundary moving following the prescribed law under the unstable operating conditions of heat exchange with the environment.

Figures 1 and 2 give the time dependences of the temperature $\theta(0, t)$ of the half-space boundary $X = 0$ under the conditions of heat exchange with the environment by the Newton law ($\mathrm{Bi}(t) \equiv \mathrm{Bi} = 1$):

under the pulsed conditions determined by the functional relationship

$$\mathrm{Bi}\,(t) = \sum_{k=0}^{2} \beta_k \left[J\,(t - t^{(k)}) - J\,(t - t^{(k+1)}) \right], \tag{30}$$

where $\{B_k\}_{k=0}^{2}$ and $\{t^{(k)}\}_{k=0}^{2}$ are the known constants and $J(t)$ is the Heaviside function [7];

under the periodic heat exchange conditions

$$\mathrm{Bi}\,(t) = 1 - \sin\,(2\pi t)\,, \quad t \geq 0\,,$$

for different laws of motion of the boundary:

at a constant velocity

$$l'(t) \equiv 1,$$

for the piecewise-constant law of motion (Fig. 1)

$$l'\,(t) = \sum_{k=0}^{2} \beta_k \left[J\,(t - t^{(k)}) - J\,(t - t^{(k+1)}) \right], \tag{31}$$

for the nonlinear law of motion (Fig. 2)

$$l'(t) = 1 - \sin(2\pi t)\,, \quad t \geq 0\,.$$

Now we consider in greater detail the evolution of the temperature profile θ(0, t) under the pulsed conditions of heat exchange (Eq. (30)) and under the conditions of heat exchange following the Newton law Bi = 1 for a constant velocity of motion $l'(t)$ = 1 of the boundary and the piecewise-constant law (31) of its motion (Fig. 1). We single out the most characteristic features of the process under study.

1. At a constant velocity of motion $l'(t)$ of the half-space boundary X = 0, the improvement in the conditions of heat exchange with the environment is accompanied by an increase in the temperature of the moving boundary (Fig. 1, curve 1).

2. Under the conditions of heat exchange following the Newton law, the rise in the velocity of motion $l'(t)$ inevitably leads to a decrease in the temperature θ(0, t) of the moving boundary (curve 2).

3. It is possible to exert a controlled influence on the temperature field of the half-space with a boundary moving following the prescribed law by means of the time control of the conditions of the heat exchange with the environment (curve 3).

It should be emphasized that many factors are responsible for the interest in the pulsed conditions of heat exchange with the piecewise-constant law Bi(t). First, along with the physical interpretation of interest, this case is important in testing the results obtained since it leads to the simplest representations of the solution of the function θ(0, t) in problem (4)–(7). Second, in this case it is possible to obtain rather simple evaluations of the asymptotic behavior of the function θ(0, t) for $t \to +\infty$ that determine the influence of the pulsed heat-exchange parameters on the stationary temperature field of the studied region. In particular, when $l'(t) \equiv 0$, these evaluations show [5] that the realization of any pulsed heat-exchange conditions with the piecewise-constant law Bi(t) does not lead to a qualitative change in the behavior of the function θ(0, t) for $t \to +\infty$: $\theta(t, 0) \to 1$ for $t \to +\infty$.

Now we consider the manner in which the mobility of the boundary affects the stationary (for $t \to +\infty$) temperature field of a region simulated by a half-space. With allowance for the above considerations we confine ourselves to evaluation of the asymptotic behavior of the function θ(X, t) for $t \to +\infty$ under the conditions of heat exchange with the environment, described by the Newton law, at a constant velocity of motion of the half-space boundary, i.e., we assume that

$$\mathrm{Bi}\,(t) \equiv \mathrm{Bi} = \mathrm{const}\,, \quad l'(t) \equiv V_0 = \mathrm{const}\,. \tag{32}$$

Taking into account Eq. (32), it is more convenient to use another representation of the solution of problem (4)–(7), which can be obtained by the method of integral Laplace transformation [7] with respect to the variable t:

$$\theta\,(X, t) = \frac{\mathrm{Bi}}{\mathrm{Bi} + V_0} \exp\left\{- V_0 X\right\} - \frac{\mathrm{Bi}}{\pi} \exp\left\{-\frac{V_0}{2}\left(\frac{V_0 t}{2} + X\right)\right\} \times$$

$$\times \int_0^\infty \frac{\exp\left\{- yt\right\}}{y + V_0^2/4} \, \frac{(\mathrm{Bi} + V_0/2)\sin(X\sqrt{y}) + \sqrt{y}\cos(X\sqrt{y})}{y + (\mathrm{Bi} + V_0/2)^2}\, dy\,.$$

From this expression we find the asymptotics for the function θ(X, t) for $t \to +\infty$:

$$\theta\,(X, t) \xrightarrow[t \to +\infty]{} \frac{\mathrm{Bi}}{\mathrm{Bi} + V_0} \exp\left\{- V_0 X\right\},$$

from which it follows that the maximum heating is achieved on the half-space boundary X = 0 and its evaluation for $t \to +\infty$ is determined by the equality

$$\lim_{t \to +\infty} \theta\,(0, t) = \frac{\mathrm{Bi}}{\mathrm{Bi} + V_0}\,.$$

Thus, the mobility of the boundary of the region simulated by the half-space results in the dependence of the temperature field formed in it on the intensity of heat transfer on the boundary of this region. At the prescribed velocity V_0 of uniform motion, this influence is the greater, the lower the intensity of the heat transfer Bi on the outer boundary.

NOTATION

x, spatial variable; $\bar{t}$, time; T, temperature; ξ, dimensionless variable; θ, dimensionless temperature; Fo, Fourier number; Bi, Biot criterion; x_*, chosen unit of scale; λ, thermal-conductivity coefficient; k, thermal-diffusivity coefficient; $\alpha = \alpha(\bar{t})$, heat-transfer coefficient; X, dimensionless variables of the moving coordinate system; $l(t)$ and $l'(t)$, law and velocity of motion of the half-space boundary in the moving coordinate system; $\theta(X, \text{Fo})$, dimensionless temperature at any point X of the half-space in the moving coordinate system; $u(t) = \theta(0, t)$, dimensionless temperature of the half-space boundary $X = 0$; $L^2[0, +\infty]$, linear space of functions quadratically integrable on a semibounded interval $[0, +\infty)$; Φ_C and Φ_S, integral operators of the cosine and sine Fourier transforms respectively; $R(t, \tau)$, resolvent; $q(\mu)$, characteristic polynomial; μ, eigenvalue of the square matrix B of order n; θ_n and I_n, zero and unit square matrices of order n; s_u, tabulation step of the function $u(t)$ in the iteration procedure. Subscripts: en, environment; 0, initial value; C and S, cosine and sine transforms.

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